Hub-activated signal transmission in complex networks

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A wide range of networked systems exhibit highly connected nodes (hubs) as prominent structural elements. The functional roles of hubs in the collective nonlinear dynamics of many such networks, however, are not well understood. Here, we propose that hubs in neural circuits may activate local signal transmission along sequences of specific subnetworks. Intriguingly, in contrast to previous suggestions of the functional roles of hubs, here, not the hubs themselves, but nonhub subnetworks transfer the signals. The core mechanism relies on hubs and nonhubs providing activating feedback to each other. It may, thus, induce the propagation of specific pulse and rate signals in neuronal and other communication networks.

Hubs—nodes that are significantly more highly connected than average—constitute a prominent structural feature of many network dynamical systems, such as infection, transportation, communication, and social networks [1]. The existence of hubs may follow from intentional design to optimize network properties (such as in airline, transportation, and technical communication infrastructure) or may emerge due to self-organization via intrinsic growth rules (World Wide Web and social networks) [1–4]. As hubs can structurally improve the capabilities of networks to transfer signals [5], it is not surprising that they were also found in the brain on different scales: In cortical neuronal circuits, hub regions are assumed to coordinate the activity of other regions and to organize the flow of information between them [6]. On the microscopic level, for instance, the nervous system of *Caenorhabditis elegans* contains single cell hubs [7] involved in the control of pheromone attraction as well as social behavior [8].

Interestingly, Bonifazi *et al*. [9] recently experimentally discovered hub cells also in higher animals where they support synchronous activity in the developing hippocampus. Yet, how exactly hubs dynamically influence information transmission in neural circuits still remains unknown [10].

In this Rapid Communication, we show that hub activity may amplify local signals and may enable their targeted transmission. Specifically, we show how hubs and nonhub subnetworks in neural circuits activate each other to exhibit synchronous pulse emission. Thereby, synchronous pulse activity may robustly propagate along sequences of nonhub subnetworks, thus, enabling directed and specific routing of information across the entire system. The generic mechanism of mutual hub and nonhub activation may equally enable the transmission of pulse-coded as well as rate-coded signals in a wide range of natural and artificial communication networks.

For an example of spiking neural circuits, consider networks of N units randomly connected to each other. Each connection is present with a fixed probability. In the simplest setting, between any pair of neurons, there is an excitatory connection of strength ϵ_+ with probability p_+ and an inhibitory connection of strength ϵ_- with probability $p_- = p_+ =: p$. The dynamics of each unit i is described by a real state variable, its membrane potential $V_i(t)$ in real time t and changes according to leaky integrate-and-fire dynamics. Specifically, V_i integrates excitatory (positive) and inhibitory (negative) pulsed inputs, and when crossing a threshold from below, the potential resets and the unit emits a pulse. This pulse arrives at the postsynaptic neurons after a transmission delay, and its effects are modeled by transient double-exponential conductance changes [11].

Typically, some of the pulse inputs to a neuron are synchronous (i.e., are received within a few milliseconds), and others are asynchronous. Whereas, the neuron integrates all inhibitory and asynchronous excitatory inputs additively, synchronous excitatory inputs are processed nonadditively (nonlinearly). This nonadditive integration takes into account the influence of fast dendritic spikes found in single neuron experiments [12] on the dendritic (input) sites of neurons: Whenever the total excitatory input to a dendrite summed over a short time interval (typically 2 to 3 ms) exceeds a dendritic threshold Θ_d, a dendritic spike is initiated and changes the membrane potential of the neuron after a short delay in a stereotypical way. We model its effect by a stereotypical current pulse causing a rapid strong increase (depolarization) of V_i, which substantially exceeds the level of depolarization expected from linear summation of inputs [11,13,14] and resembles the shape of the depolarization found in experiments [12]. We account for the experimentally observed saturation of the depolarization by inputs exceeding the dendritic threshold Θ_d [12] as well as for the refractory time of ion channels generating dendritic spikes by assuming that the dendrite becomes refractory for a short time period $t^{\mathrm{ref,ds}}$ after a dendritic spike is initiated.

In our numerical simulations, we focus on networks of spiking leaky integrate-and-fire neurons as described above (Simulation results were obtained using the simulation software NEST [15]). To achieve a mechanistic understanding of the observed phenomena, we further derive an analytically tractable description in terms of probabilistic threshold units below.

Motivated by recent anatomical and physiological findings [9], we assume that some $N_h \geqslant 0$ neurons are hub neurons. They are distinguished (exclusively) by an increased

probability $p_h > p$ to receive input connections from other units in the network.

Following a standard approach for signal transmission in cortical networks [16], we consider signal propagation along weak feed-forward structures: The network contains sequences (chains) of m subnetworks (groups) with N_g neurons each. The neurons in each group are randomly chosen from the nonhub population, and excitatory connection strengths between subsequent subnetworks are increased compared to other coupling strengths in the network, $\epsilon_{sub} > \epsilon_+$.

We consider networks with balanced excitatory and inhibitory connectivities such that, in the absence of external inputs, asynchronous irregular spiking dynamics constitutes their ground state activity [17]. Externally exciting an initial subnetwork to spike synchronously causes synchronous inputs to neurons of the downstream subnetwork and induces synchronous spiking of a fraction of its neurons. This may excite neurons in the ensuing subnetwork to spike, etc., thereby transmitting signals along the chain of subnetworks. However, as the subnetworks are parts of a larger recurrent network, synchronous activity not only may spread from one subnetwork to the next, but also may induce a synchronous spiking response (echo) in the remainder of the network. Depending on the parameters and the number of initially synchronous neurons g_0 in the first subnetwork and r_0 in the remainder of the network, synchronous activity may, in principle, stably propagate, spread across the entire recurrent network, and, thus, may obscure a propagation signal (not shown) or may extinguish after a few subnetworks.

Sample simulations of networks without hubs [$N_h = 0$, Fig. 1(a)] illustrate that spreading and dying out of synchrony dominate state space in agreement with the literature [18] because there is no mechanism keeping the synchronization in the network remainder at a moderate level.

Networks with a substantial number N_h of hub units exhibit qualitatively different dynamics and support signal transmission: As hubs receive more input connections than other units, they have a higher probability of spiking in response to synchronous inputs from a certain subnetwork. Thereby, hubs may establish a synchronous response to propagating synchronous pulses. Due to increased connectivity at hubs only, such an echo is confined to the hub neuron subpopulation and, thus, does not spread over the entire network [cf. Fig. 1(b)].

The increased connectivity towards hubs plays an interesting double role: It ensures that a population of sufficiently many hub neurons exhibits, itself, synchronous activity if supported by synchrony in a (nonhub) subnetwork. At the same time, the fact that the network remainder without hubs has relatively low connectivity prevents spreading of synchronous activity beyond the hub population. This combination enables robust synchrony propagation along sequences of nonhub subnetworks for a range of initially synchronous neurons g_0 in a subnetwork [cf. Fig. 1(b), main panel].

To further understand this coaction mechanism, we consider the dynamics only at the relevant time intervals where synchronous pulses are sent and are received. Observing that
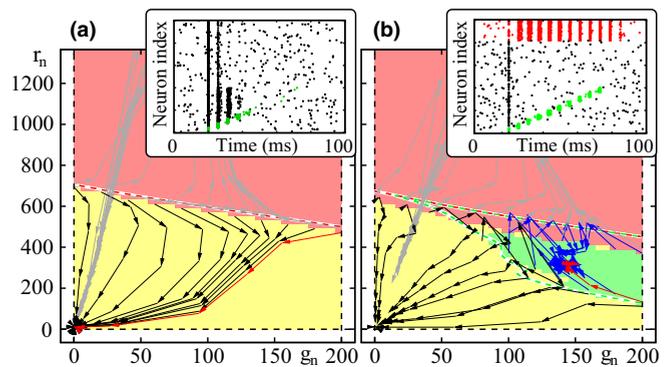
FIG. 1. (Color online) Hubs activate signal transmission in a neural network. Signals consist of localized synchronous spiking activity (times marked green in the insets) transmitted across a sequence of subnetworks (displayed as lowest neuron indices). Spike times of hubs (red) displayed at the top, above those of the remaining neurons (black). Main panels: Joint dynamics of the number of synchronously spiking neurons in the nth subnetwork (g_n) and the total number of synchronously coactivated neurons of the network remainder (r_n) during signal propagation initiated by synchronously stimulating g_0 neurons of the initial subnetwork and r_0 neurons of the network remainder. (a) In networks without hubs, the overall network activity either becomes pathological (large scale synchrony: red shading, gray trajectories) or extinguishes quickly to background activity (yellow shading, black trajectories). Hub neurons in otherwise the same network (b) can induce a persistent signal transmission across nonhubs (green shading, blue trajectories) by generating sustained but bounded synchrony. Red trajectories indicate example dynamics shown in the insets. The dashed lines indicate the borders of activity regions analytically estimated in this Rapid Communication (cf. Eqs. (5) and (6) and Ref. [11]). Parameters: $N = 5000$, $m = 10$, $N_g = 200$, and $p = 0.05$; furthermore, $N_h = 0$ in (a) and $N_h = 900$ and $p_h = 0.12$ in (b).

the neurons effectively act as probabilistic threshold units, we derive an approximate analytic map for the joint response sizes of active hubs and signal carrying (nonhub) units. The spiking probability due to a synchronous input below the dendritic threshold Θ_d is very low [cf. Fig. 2(a)] so that we neglect it against the probability of spiking due to inputs above threshold. The probability $p^{sp}(I_+,I_-)$ of a neuron spiking in response to excitatory and inhibitory inputs I_+ and I_- is a function of the probability distribution of the membrane potentials of that neuron at the time of input reception. We take this dependency into account by assuming that, immediately before every spike reception time, the neuronal state is distributed as in the unperturbed ground state. The function p^{sp}, thus, obeys

$$p^{sp}(I_+,I_-) = \begin{cases} 0 & \text{for} \quad I_+ < \Theta_d, \\ p^0(I_-) & \text{for} \quad I_+ \geqslant \Theta_d, \end{cases} \quad (1)$$

where $p^0(I_-)$ is the spiking probability of a neuron in the ground state receiving a dendritically suprathreshold excitatory input and an inhibitory input of size I_-. In particular, $p^0(0)$ is the spiking probability of a single neuron when a dendritic spike is generated in the absence of inhibition. p^0 depends solely on the inhibitory input I_- because, on one
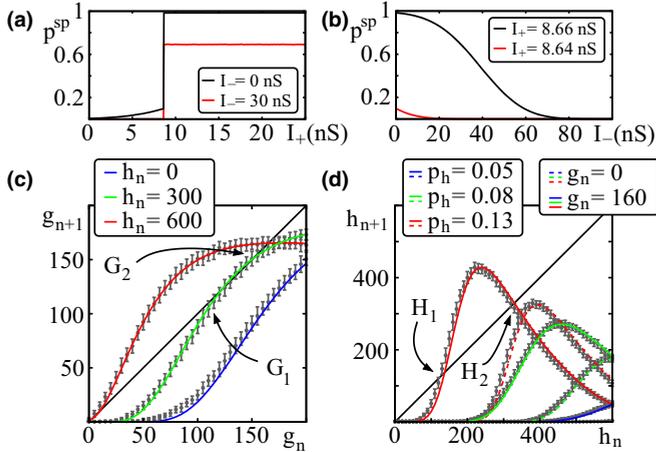
FIG. 2. (Color online) Hubs induce tangent bifurcations towards signal transmission (neuron and network parameters as in Fig. 1). (a) and (b) Firing probability $p^{\rm sp}$ of a neuron in the ground state as a function of synchronous (a) excitatory input I_+ and (b) inhibitory input I_-. (c) and (d) Iterated maps for (c) the number g_n of (synchronously) active neurons in the nth subnetwork (different colors indicate different fixed h_n's) and (d) the number of synchronized hub neurons h_n (different colors: p_h fixed; different line styles: g_n fixed). Analytical predictions [solid and dashed lines; Eqs. (5) and (6)] agree well with numerical simulations of the spiking neural network model (markers). Sufficiently large h_n enables propagation of synchrony (c), and a sufficiently large connection probability p_h enables a persistent hub echo for a propagating synchronous pulse (d). Hubs and nonhubs reactivate each other.

hand, only sufficiently strong excitatory inputs exceeding the dendritic threshold elicit a dendritic spike and the effect of a dendritic spike on the postsynaptic neuron saturates, i.e., it remains the same, for stronger excitation [cf. Fig. 2(a)] as found in experiments [12]. On the other hand, inhibition will generally decrease a neuron's spiking probability as it partially compensates the input to the soma due to the dendritic spike (cf. Fig. 2(b) and the experimental findings in Ref. [19]). The precise form of $p^0(I_-)$ depends on the details of the background activity and the properties of neurons and interactions. As will become clear below, all *qualitatively* similar $p^0(I_-)$'s induce the same type of bifurcation relevant for robust signal transmission, and thus, details of $p^0(I_-)$ do not matter.

During robust signal transmission promoted by a hub echo, spikes of hub neurons and neurons of the currently active subnetwork dominate the network dynamics [cf. inset of Fig. 1(b)]. We, thus, focus on these two groups of neurons. The influence of the remaining neurons can be analytically derived analogously [11]. To be specific, assume that $g_n \leqslant N_g$ neurons in a given subnetwork n and $h_n \leqslant N_h$ hub neurons are active simultaneously, i.e., they spike synchronously. Given the random network topology, for sufficiently large g_n and h_n, the total input to the neurons of the $(n + 1)$th subnetwork is approximately Gaussian distributed (approximating the actual binomial distributions) $I_\pm \sim \mathcal{N}(\mu_\pm, \sigma_\pm^2)$ with probability density functions $f_+(I_+)$ and $f_-(I_-)$ and means and variances

given by

$$\mu_+ = (\epsilon_+ h_n + \epsilon_{\rm c} g_n)p, \quad \sigma_+^2 = \left(\epsilon_+^2 h_n + \epsilon_{\rm c}^2 g_n\right)p(1 - p), \tag{2}$$

$$\mu_- = \epsilon_-(h_n + g_n)p \quad \text{and} \quad \sigma_-^2 = \epsilon_-^2(h_n + g_n)p(1 - p). \tag{3}$$

The expected number of neurons that spike synchronously in subnetwork $n + 1$ becomes

$$g_{n+1} = N_g \int_0^\infty \int_0^\infty p^{\rm sp}(I_+, I_-)f_+(I_+)f_-(I_-)dI_+ dI_-. \tag{4}$$

Whereas, $p^{\rm sp}$ discontinuously depends on I_+, it changes smoothly and, thus, locally linearly with I_- [cf. Figs. 2(a) and 2(b)] such that we may set $f_-(I_-) = \delta(I_- - \mu_-)$ to evaluate the integral in Eq. (4), yielding the iterated map,

$$g_{n+1} = N_g p^0(\mu_-)\frac{1}{2}\left(1 + {\rm Erf}\left[\frac{\Theta_{\rm d} - \mu_+}{\sqrt{2}\sigma_+}\right]\right) \tag{5}$$

for the number of active signal transferring (nonhub) neurons in the next subnetwork. Note that all three quantities μ_-, μ_+, and σ_+ depend on h_n and g_n through Eqs. (2) and (3).

The iterated map for the number of synchronously active hub neurons h_{n+1} is derived analogously: We discard those h_n neurons that have spiked together with the nth subnetwork because they are unlikely to spike again due to their relative refractoriness such that $N_h - h_n$ hub neurons are available to spike. Replacing N_g by $N_h - h_n$ in Eq. (4) and computing the Gaussian probability densities of the inputs yield the iterated map,

$$h_{n+1} = (N_h - h_n)p^0(\tilde{\mu}_-)\frac{1}{2}\left(1 + {\rm Erf}\left[\frac{\Theta_{\rm d} - \tilde{\mu}_+}{\sqrt{2}\tilde{\sigma}_+}\right]\right), \tag{6}$$

where $\tilde{\mu}_+ = \epsilon_+ p_h(h_n + g_n)$, $\tilde{\mu}_- = \epsilon_- p_h(h_n + g_n)$, and $\tilde{\sigma}_+^2 = \epsilon_+^2 p_h(1 - p_h)(h_n + g_n)$.

The joint two-dimensional map (5) and (6) explicates how the hub neurons can enable robust propagation of synchrony [see Figs. 2(c) and 2(d)]: For a given number h_n of active hub neurons, the fixed points of Eq. (5) determine whether robust propagation of synchrony can be initiated in the chain of subnetworks. For networks without (active) hubs $h_n = 0$, there is only one fixed point $G_0 = 0$, and any initial synchronous pulse extinguishes after a small number of subnetworks. With increasing h_n, two additional fixed points G_1 (unstable) and G_2 (stable) appear via a tangent bifurcation at some $h_n = h^*$, and robust signal transmission is enabled for initial synchronous pulses $g_0 \geqslant G_1$ [cf. Fig. 2(c)]. For large numbers of active hubs, even small initial group sizes g_0 are sufficient to generate robust signal transmission across the chain of subnetworks.

Analogously, the fixed points of Eq. (6) determine whether a persistent hub echo for the propagating synchronous pulse establishes for a given hub connectivity p_h and group size g_n [cf. Fig. 2(d)]. For small p_h and g_n, there is only one fixed point $H_0 = 0$. With increasing p_h or g_n, two additional fixed points H_1 (unstable) and H_2 (stable) appear via a tangent bifurcation for some p_h^* and g^*. Thus, for sufficiently large hub connectivity $p_h \geqslant p_h^*$, a persistent echo for a propagating
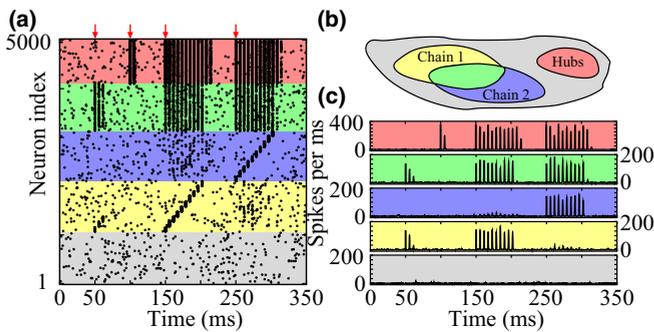
FIG. 3. (Color online) Hub neurons act as a generic signal ampli-fier and activate different signal routes. The figure shows simulation data for a sparse recurrent spiking neural network [the same network as in Fig. 1(b)] with two chains of subnetworks. (a) Raster plot of the network activity; the background colors indicate whether the neurons are members of one or both chains, hub neurons or remaining neurons, as visualized by (b). (c) Current activity (spikes per bin; bin size 1 ms) of the different neuron populations. If synchronous spiking is initiated either in the initial subnetwork of one chain ($t = 50$ ms) or in the hub neurons ($t = 100$ ms) only, synchronous activity extinguishes quickly. In contrast, if the initial subnetwork of one of the chains as well as the hub neurons are excited ($t = 150$ and 250 ms), robust propagation of synchrony establishes in that specific chain.

synchronous pulse of size g_n can be established; equivalently, for fixed connectivity p_h, sufficiently many synchronously active neurons in the subnetwork maintain a hub echo. The bifurcations resulting from the analytic mapping (5) and (6) approximately predict the numerically found region where robust signal transmission is possible (see the dashed line in Fig. 1 and Ref. [11]).

Having gained this mechanistic understanding, we now illustrate that hubs unspecifically but selectively activate synchrony propagation. Signal propagation becomes possible along any chain of subnetworks that structurally exists in the system if its initial group is excited. In particular, in systems with a second chain of subnetworks embedded, the mutual hub and nonhub feedback can amplify signal transmission along one chain without activating transmission in the other one (cf. Fig. 3).

To summarize, we have demonstrated that hubs may act as amplifiers that enable signal generation and transmission in recurrent networks. So far, hubs were thought to directly distribute various types of signals (e.g., actual information on the World Wide Web, certain infections in disease spreading, and people in travel networks) across a network. We now identified a complementary fundamentally different role of hubs in signal transmission: The hubs studied here do not communicate the specific signal themselves; instead, increased hub activity mirrors the presence of some localized signal in other network parts, and the hubs promote the transmission of any such signal across sequences of nonhub subnetworks.

This mechanism of hub-activated signal propagation essentially relies on: (a) the existence of some highly connected nodes and (b) some sharp thresholdlike processing of incoming inputs by single units (as, for instance, mediated by fast dendritic sodium spikes in neural circuits).

Furthermore, the phenomenon is robust against changes in the network topology. As an explicit example, we show that it occurs in scale-free networks [1] where hubs naturally emerge due to the "fat tail" of the degree distribution (cf. Ref. [11] for an example). We, thus, expect that this type of signal transmission may well play a role in biological networks and may even be exploited in self-organized solutions of technical communication networks [20].

It has long been hypothesized that cortical neural networks transmit signals via propagating synchronous spiking activity across subnetworks connected in a feed-forward manner [14,16,18]. The results above now suggest that hubs might enable robust propagation of synchronous signals even in weak embedded feed-forward structures by echoing the synchronous signal propagating along them. In the absence of hubs (and due to the lack of a confining mechanism), the echo cannot contribute in this way as synchronous activity either dies out or spreads across the whole network and causes pathological activity [e.g., Ref. [18] and cf. also Fig. 1(a)]. To reveal the essential mechanisms underlying signal transmission, we disregarded "Dale's law" [21] (stating that each neuron either has only excitatory or has only inhibitory outgoing connections) and considered a simple bimodal degree distribution clearly splitting the system into hub and nonhub neurons. In additional simulations, we verified that the uncovered new type of signal transmission equally emerges in networks with neurons obeying Dale's law and exhibiting natural and broad degree distributions [11].

Interestingly, hubs have recently also been uncovered experimentally in the developing hippocampus [9]. As in the adult hippocampus, synchronized oscillatory activity abounds, and the structural feature of hub neurons might support the directed transmission of specific signals. Such hub-feedback support may provide one reason why hubs emerge in these systems in the first place, cf. also Ref. [22].

Specifically, hub feedback might also be involved in the replay of spike sequences during so-called sharp wave-ripple complexes observed in the hippocampus [23]. Here, during sleep, neurons are activated in the same order as they have been during an exploration phase, accompanied by strong network oscillations. Whereas, most neurons take part in only a few of the different replayed patterns, some are activated in a large fraction of events [24]. Our results suggest that the latter may be unspecific for certain memories and, rather, hub neurons generating a synchronous feedback signal to stabilize signal propagation along a previously learned feed-forward structure of specific neurons.

Finally, our analytical results (5) and (6) for the activity of the hubs and the signal-carrying units clearly demonstrate that the principle of mutual activation underlying the support of signal transmission may act in any network of sharply nonlinear (probabilistic) threshold units as characterizing, e.g., transmission of rate activities in networks of neural populations (McCullogh-Pitts model, e.g., Ref. [25]), (failure) cascades in social, supply, or communication networks (e.g., Ref. [26]), or signaling in gene and protein networks (threshold Boolean networks, e.g., Ref. [27]).

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