Outage Analysis for Decode-and-Forward Multirelay Systems Allowing Intra-Link Errors

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***Abstract*—In the context of distributed source coding, the so-called many-help-one problem comprises a set of auxiliary sources aimed at helping the decoder retrieve a primary source. In this letter, we build on the newly derived admissible rate region of a certain binary many-help-one problem to obtain the outage probability (OP) of a decode-and-forward (DF) multirelay system that allows intra-link errors (IE) to be forwarded to the destination. In addition, for comparison, we obtain the OP of a conventional DF multirelay system, which discards any IEs. We show that, the more relays are employed, the more advantageous it is to forward the IEs, as opposed to discarding them.**

***Index Terms*—Decode-and-forward, distributed source coding, many-help-one problem, Slepian-Wolf theorem.**

I. Introduction

Fifth-generation mobile networks will face several challenges to cope with all emerging application scenarios [1]. One goal is to provide connectivity everywhere, including highways and remote low-populated areas. Providing internet service in remote areas demands a large cell, and the link budget for the uplink severely limits the throughput for devices on the cell edge. Highway coverage is also challenging, especially in locations away from urban areas. The high speed of the devices and the rich propagation environment lead to a doubly-dispersive channel. In both scenarios, device-to-device communications based on decode-and-forward (DF) relaying schemes is a promising solution to ensure reliability and data rates at acceptable levels.

In *conventional* DF, the decoded message is discarded by the relay whenever an error is detected [2]. On the other hand, by allowing such *intra-link errors* (IE) to be forwarded to the destination, an improved end-to-end performance can be achieved, as shown in [3] for a classic topology with one source, one relay, and one destination (the notion of allowable intra-link errors was first introduced in [4]). The central idea is that an erroneous relay message is still somewhat correlated to the source message, thus serving as valuable side information in the decoding process at the destination. Based on the theorems of source coding with side information [5] and source-channel separation [6, Th. 3.7], the outage probability (OP) of a DF-IE relaying system was derived in [7] for a classic three-node topology, assuming that all the links undergo block Rayleigh fading. In that work, DF-IE was shown to outperform conventional DF, analyzed in [2].

One may want to extend the outage analysis in [7] to two or more relays. This is an important extension, because in the harsh environment of potential applications (e.g., fifth-generation vehicular networks) parallel routes (i.e., multiple relays) may be required to keep connectivity at an acceptable level. In principle, such an extended analysis could be attained based on the admissible rate region for the distributed encoding of one primary source and multiple auxiliary ones. This is the so-called many-help-one problem, which is still open in its general form. Recently, this problem has been solved for the special case of binary sources, Hamming distortion measure, and independently degraded helpers (i.e., conditionally independent auxiliary sources, given the primary source) [8].

In this work we capitalize on [8] to extend the outage analysis of DF-IE in [7] to an arbitrary number of relays. For comparison, we also derive the OP of conventional DF with multiple relays. A conventional DF system was analyzed in [2], but therein maximal-ratio combining (MRC) was assumed at the destination, which is a suboptimal combining scheme. Herein we assume joint decoding instead, which is optimal [9]. We derive OP expressions in exact integral form. For conventional DF we also derive an asymptotic OP expression at high signal-to-noise ratio (SNR). Finally, for DF-IE we derive the amount of extra channel usage required by the forwarding of erroneous messages. Our results reveal that DF-IE not only outperforms conventional DF, but also becomes more advantageous as the number of relays increases.

In what follows, the alphabet of a random variable (RV) X with sample value x and probability mass function $p(x)$ is denoted by $\mathcal{X}$, and its cardinality, by $|\mathcal{X}|$. Also, $\mathbf{X}$ and $\mathbf{x}$ represent n-sample sequences of X and x, respectively, n being the length of a transmission block. We use k to denote time, i to identify a node, and calligraphic letters to denote sets. The mth element of a set $\mathcal{Y}$ is denoted by y_m, $m \in \{1, \ldots, |\mathcal{Y}|\}$. Moreover, we define the following: $A_{\mathcal{S}} \triangleq \{A_i | i \in \mathcal{S}\}$ is an indexed series of RVs, $\mathcal{N} \triangleq \{1, 2, ..., N\}$, $\mathcal{L} \triangleq \{2, 3, ..., N\}$, $h(\cdot)$ is the binary entropy function, $a_1 * a_2 \triangleq a_1(1-a_2)+(1-a_1)a_2$ is the

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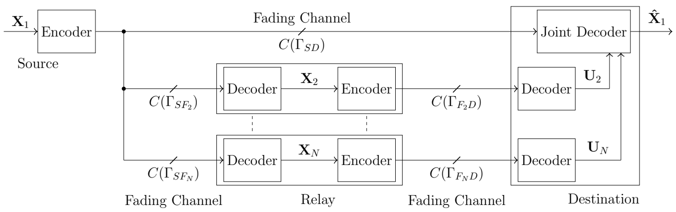

Fig. 1: Decode-and-forward multirelay system allowing intra-link errors.

binary convolution, $a_1 * ... * a_N \triangleq a_1 * (... * (a_{N-1} * a_N)...)$ is the multivariate binary convolution, and $\approx$ means "asymptotically equal to."

II. System Model

We consider a dual-hop relaying system in which one source (S) cooperates with $N-1$ half-duplex DF-IE relays ($F_i, \forall i \in \mathcal{L}$) to transmit information to one destination (D), as shown in Fig. 1. We assume that all the channels undergo independent Rayleigh fading and additive white Gaussian noise with mean power N_0. The probability density function (pdf) of the instantaneous received SNR of all the links is then exponentially distributed, given by

$$f_{\Gamma_\nu}(\gamma) = {}^1\!/\bar{\Gamma}_\nu \cdot \exp\left(-{}^\gamma\!/\bar{\Gamma}_\nu\right), \tag{1}$$

where $\nu \in \{SD, SF_2, \ldots, SF_N, F_2D, \ldots, F_ND\}$, $\bar{\Gamma}_\nu$ is the mean value of Γ_ν, $\Gamma_{SD} \triangleq |h_{SD}|^2 P_S/N_0$ is the direct-link received SNR from S to D, $\Gamma_{SF_i} \triangleq |h_{SF_i}|^2 P_S/N_0$ is the first-hop received SNR from S to F_i, and $\Gamma_{F_iD} \triangleq |h_{F_iD}|^2 P_{F_i}/N_0$ is the second-hop received SNR from F_i to D. In these expressions, $|h_{SD}|^2$, $|h_{SF_i}|^2$, and $|h_{F_iD}|^2$ denote the random channel power gains of the corresponding links, and P_S and P_{F_i} stand for the transmit powers at S and F_i, respectively. The channel state information is known at the receiver.

We consider a binary memoryless source, the output sequence of which is $\{X_{1,k}\}_{k=1}^{\infty}$, with k denoting discrete time. The n-sample source sequence shall be represented in vector form as $\mathbf{X}_1 = [X_{1,1}, X_{1,2}, ..., X_{1,n}]$. When appropriate, for simplicity, we shall drop the temporal index, denoting the source output simply as X_1. The sequence $\mathbf{X}_1$ takes values from a binary set $\mathcal{X} = \{0, 1\}$ with uniform probabilities, i.e., $\Pr[X_1 = 0] = \Pr[X_1 = 1] = 0.5$.

The relaying system operates on a time-division multiple access basis, with the transmission process being accomplished in three steps, as follows. First, S encodes, modulates, and broadcasts $\mathbf{X}_1$ to D and all F_i's. Second, each F_i demodulates, decodes, interleaves, re-encodes, modulates, and forwards to D the received source sequence. The decoded relay sequences $\mathbf{X}_i, \forall i \in \mathcal{L}$, differ from the original source sequence $\mathbf{X}_1$ according to certain first-hop error probabilities $p_i \triangleq \Pr[X_i \neq X_1]$, which depend on the instantaneous received SNRs between S and F_i, Γ_{SF_i}. Lastly, all received relay sequences are independently demodulated and decoded at D. Again, the decoded destination sequences $\mathbf{U}_i, \forall i \in \mathcal{L}$, differ from the relay sequences $\mathbf{X}_i$ according to certain second-hop error probabilities $\alpha_i \triangleq \Pr[U_i \neq X_i]$, which depend on the instantaneous received SNRs between F_i and D, Γ_{F_iD}. The decoded sequences $\mathbf{U}_i, \forall i \in \mathcal{L}$, and the via the direct-link received source sequence are jointly decoded at D to retrieve $\mathbf{X}_1$. Such a joint decoding scheme is able to exploit the correlation among the received sequences in an optimal manner [8].

III. Preliminaries

In this section we outline the connection between the newly derived admissible rate region of a binary many-help-one problem with independently degraded helpers [8] and the outage analysis of DF-IE multirelay systems. As mentioned before, the many-help-one problem is that of a given number of auxiliary sources aimed to provide side information to a decoder for the perfect reconstruction of a primary source. Next, for convenience, we reproduce the referred result from [8].

[8, Theorem 8]: *If X_1 is a binary, uniformly distributed, primary source, $X_{\mathcal{L}}$ are binary auxiliary sources with joint pmf $p(x_1, x_{\mathcal{L}}) = p(x_1)\prod_{i=2}^{N} p(x_i|x_1)$, and $U_{\mathcal{L}}$ are binary RVs with pmf $p(u_i, x_i) = p(x_i)p(u_i|x_i), i \in \mathcal{L}$, then the admissible rate region $\mathcal{R}_{DF\text{-}IE}$ is the set of all rate N-tuples $R_{\mathcal{N}}$ for which the following conditions are satisfied:*

$$\begin{cases} R_1 \geq \psi(p_{\mathcal{L}}, \alpha_{\mathcal{L}}), & \text{(2a)} \\ R_i \geq 1 - h(\alpha_i), & \text{(2b)} \end{cases}$$

for $0 \leq \alpha_i \leq 0.5$ and fixed $p_i, \forall i \in \mathcal{L}$, where

$$\psi(p_{\mathcal{L}}, \alpha_{\mathcal{L}}) \triangleq \sum_{l_1 \in \mathcal{L}} h\left(p_{l_1} * \alpha_{l_1}\right) - \sum_{\{l_1, l_2\} \in \mathcal{L}} h\left(p_{l_1} * \alpha_{l_1} * p_{l_2} * \alpha_{l_2}\right) + ... \pm \sum_{\{l_1, \ldots, l_{|\mathcal{L}|}\} \in \mathcal{L}} h\left(p_{l_1} * \alpha_{l_1} * ... * p_{l_{|\mathcal{L}|}} * \alpha_{l_{|\mathcal{L}|}}\right); \tag{3}$$

and for which there exists a function $g(\cdot)$ such that $\mathbb{E}\left[d_H(X_1, g(X_1, U_{\mathcal{L}}))\right] \leq \epsilon$, with $d_H(\cdot,\cdot)$ being the Hamming distance.

In the above theorem, the auxiliary sources are assumed to be conditionally independent given the primary source. Note that this assumption perfectly matches the DF-IE system depicted in Fig. 1, in which the decoded relay sequences are independently degraded replicas of the source sequence.

Now we map the rate constraints into corresponding SNR constraints. Due to the low-latency requirements in 5G, relatively short encoded sequences are envisaged. So we can consider that the length of a fading block is greater than the transmission interval of each encoded sequence. Based on this block Rayleigh fading assumption, the maximum achievable value of the transmission rate R_i for each relay is related to the second-hop received SNR Γ_{F_iD} by [6, Th. 3.7]

$$R_i = {}^1\!/R_{i,\mathrm{c}} \cdot \phi\left(\Gamma_{F_iD}\right), \tag{4}$$

where $R_{i,\mathrm{c}}$ represents the spectral efficiency, measured in information bits per channel symbol, and $\phi(\Gamma_{F_iD}) \triangleq \log_2(1 + \Gamma_{F_iD})$ is the AWGN channel capacity. For a modulation scheme with M symbols and a channel code rate of $R_{i,\mathrm{cod}}$, it follows that $R_{i,\mathrm{c}} = R_{i,\mathrm{cod}} \log_2 M$. Herein, for simplicity, we assume $R_{i,\mathrm{c}} = R_\mathrm{c}, \forall i \in \mathcal{L}$. A relationship similar to (4) holds true for the direct link of the first hops.

In (2a)–(2b), two kinds of error probabilities occur, namely, the source-relay error probabilities $p_i = \Pr[X_1 \neq X_i]$ and the relay-destination error probabilities $\alpha_i = \Pr[X_i \neq U_i]$, $i \in \mathcal{L}$. In both cases the received sequences of $\mathbf{X}_i$ and $\mathbf{U}_i$, $\forall i \in \mathcal{L}$, are decoded independently from each other, as in the

DF-IE system model. From the assumption of block Rayleigh fading, the error probabilities are constant over one frame, but vary from frame to frame. There exists a lower bound $\xi(\cdot)$ on the achievable error probability which can be related to the corresponding instantaneous received SNR by means of [7]

$$b_i \geq \xi(a_i) = \begin{cases} h^{-1}\left(1 - {\phi(a_i)}/{R_\mathrm{c}}\right) & \text{for } 0 \leq a_i < A_1, \\ 0 & \text{for } a_i \geq A_1, \end{cases} \quad (5)$$

where $(a_i, b_i) \in \{(\Gamma_{F_iD}, \alpha_i), (\Gamma_{SF_i}, p_i)\}$, $i \in \mathcal{L}$, and $A_1 \triangleq 2^{R_\mathrm{c}} - 1$. The achievability of the p_i lower bound depends on the existence of a code which is capacity-achieving for the direct and source-relay links at the same time. The following derivation is based on the error probability lower bound, i.e., $b_i = \xi(a_i)$ in (5). Note that p_i and α_i are RVs.

IV. Outage Probability of DF-IE

An outage event occurs whenever the transmission rates $R_1, ..., R_N$ fall outside the admissible rate region $\mathcal{R}_{\text{DF-IE}}$ defined in (2a) and (2b). Using (4), these rate constraints can be mapped into a set of equivalent SNR constraints. The OP of DF-IE, $P^{\text{out}}_{\text{DF-IE},N}$, is then assessed as follows: (i) for a given realization of the relaying-link received SNRs, $\Gamma_{SF_\mathcal{L}}$ and $\Gamma_{F_\mathcal{L}D}$, we compute the probability that the maximum achievable value of the direct-link transmission rate, $({1}/{R_\mathrm{c}})\,\phi(\Gamma_{SD})$, does not fulfill the constraint in (2a), with the error probabilities $p_i(\Gamma_{SF_i})$ and $\alpha_i(\Gamma_{F_iD})$ being given as in (5); (ii) we sum over all possible realizations of $\Gamma_{SF_\mathcal{L}}$ and $\Gamma_{F_\mathcal{L}D}$. Thus, we have

$$\begin{aligned} P^{\text{out}}_{\text{DF-IE},N} &= \Pr\big[0 \leq R_1 < \psi(p_\mathcal{L}, \alpha_\mathcal{L}), \\ &\qquad \{0 \leq p_i \leq 0.5, 0 \leq \alpha_i \leq 0.5, \forall i \in \mathcal{L}\}\big] \quad (6) \\ &= \Pr\big[0 \leq \phi(\Gamma_{SD}) < R_\mathrm{c} \cdot \psi(p_\mathcal{L}, \alpha_\mathcal{L}), \\ &\qquad \{0 \leq \xi(\Gamma_{SF_i}) \leq 0.5, 0 \leq \xi(\Gamma_{F_iD}) \leq 0.5, \forall i \in \mathcal{L}\}\big] \quad (7) \\ &= \Pr\Big[0 \leq \Gamma_{SD} < 2^{R_\mathrm{c} \cdot \psi\left[\{\xi(\Gamma_{SF_i}), \forall i \in \mathcal{L}\}, \{\xi(\Gamma_{F_iD}), \forall i \in \mathcal{L}\}\right]} - 1, \\ &\qquad \{0 \leq \Gamma_{SF_i} \leq \infty, 0 \leq \Gamma_{F_iD} \leq \infty, \forall i \in \mathcal{L}\}\Big] \quad (8) \\ &= \int_0^\infty \cdots \int_0^\infty \int_0^\infty \cdots \int_0^\infty \left\{1 - \exp\left[-\left(2^{R_\mathrm{c}\psi[\cdot]} - 1\right)/\bar{\Gamma}_{SD}\right]\right\} \\ &\qquad \prod_{i=2}^N f_{\Gamma_{SF_i}}(\gamma_{SF_i}) f_{\Gamma_{F_iD}}(\gamma_{F_iD}) \\ &\qquad \mathrm{d}\gamma_{SF_N} \ldots \mathrm{d}\gamma_{SF_2} \mathrm{d}\gamma_{F_ND} \ldots \mathrm{d}\gamma_{F_2D}. \quad (9) \end{aligned}$$

The above steps are justified as follows: (6) is the rate constraint in (2a) in terms of the error probabilities $p_\mathcal{L}$ and $\alpha_\mathcal{L}$; in (7), the rate constraint is mapped into an SNR constraint with use of (4), and the error probabilities $p_\mathcal{L}$ and $\alpha_\mathcal{L}$ are written in terms of $\Gamma_{SF_\mathcal{L}}$ and $\Gamma_{F_\mathcal{L}D}$, based on (5); in (8), all events are expressed directly in terms of SNR; in (9), the OP is established in integral form, from the assumption that all received SNRs are independent and follow (1). Apart from the integration over γ_{SD}, the expression in (9) cannot be solved in closed form, thus requiring numerical evaluation.

V. Outage Probability of Conventional DF

In this section we derive the OP of a conventional DF multirelay scheme. Unlike [2], instead of MRC, we assume joint decoding at the destination, which is optimal.

In conventional DF, a relay forwards a sequence iff it is detected error free. The corresponding OP can be assessed in three stages: (1) we calculate the probability that the destination fails to recover the source sequence when received by the direct link and forwarded by a given subset of relays F_i, $\forall i \in \mathcal{S} \subseteq \mathcal{L}$; (2) we calculate the probability that the source sequence be forwarded by each such subset; and (3) we sum over the probabilities of all outage events, i.e., $\forall \mathcal{S} \subseteq \mathcal{L}$. Next we elaborate on these stages.

Stage (1): Slepian and Wolf [10] considered a source coding problem where the decoder aims at perfectly reproducing correlated sources that are separately encoded at different terminals. The Slepian-Wolf admissible rate region is suitable for conventional DF. With the assumption that a given subset of the relays $F_i, \forall i \in \mathcal{S} \subseteq \mathcal{L}$, have perfectly reconstructed the source sequence, the Slepian-Wolf admissible rate region for $1 + |\mathcal{S}|$ *identical* "sources" $X_1, X_{s_1}, ..., X_{s_{|\mathcal{S}|}}$ simplifies to

$$\textstyle\sum_{i \in \{1\} \cup \mathcal{S}} R_i \geq H(X_1) = 1. \quad (10)$$

The set of $(1 + |\mathcal{S}|)$-tuples $R_1, R_{s_1}, ..., R_{s_{|\mathcal{S}|}}$ that satisfy the constraint in (10) is referred to as the DF admissible rate region. The corresponding OP was derived in [9, Sec. IV-A] in the context of multi-connectivity, as

$$\begin{aligned} P^{\text{out}}_{\text{DF},\mathcal{S}} &\triangleq \Pr\left[0 \leq R_1 + R_{s_1} + \ldots + R_{s_{|\mathcal{S}|}} < 1\right] \quad (11) \\ &= \Pr\Big[0 \leq \Gamma_{SD} < 2^{R_\mathrm{c}} - 1, 0 \leq \Gamma_{F_{s_1}D} < 2^{R_\mathrm{c} - \phi(\Gamma_{SD})} - 1, \\ &\quad \ldots, 0 \leq \Gamma_{F_{s_{|\mathcal{S}|}}D} < 2^{R_\mathrm{c} - \phi(\Gamma_{SD}) - \ldots - \phi(\Gamma_{F_{s_{|\mathcal{S}|-1}}D})} - 1\Big] \quad (12) \\ &= \int_{\gamma_{SD}=0}^{2^{R_\mathrm{c}}-1} \int_{\gamma_{F_{s_1}D}=0}^{2^{R_\mathrm{c} - \phi(\gamma_{SD})}-1} \cdots \int_{\gamma_{F_{s_{|\mathcal{S}|}}D}=0}^{2^{R_\mathrm{c} - \phi(\gamma_{SD}) - \ldots - \phi(\gamma_{F_{s_{|\mathcal{S}|-1}}D})}-1} \\ &\qquad f_{\Gamma_{SD}}(\gamma_{SD}) f_{\Gamma_{F_{s_1}D}}(\gamma_{F_{s_1}D}) \ldots f_{\Gamma_{F_{s_{|\mathcal{S}|}}D}}(\gamma_{F_{s_{|\mathcal{S}|}}D}) \\ &\qquad \mathrm{d}\gamma_{F_{s_{|\mathcal{S}|}}D} \ldots \mathrm{d}\gamma_{F_{s_1}D} \mathrm{d}\gamma_{SD}. \quad (13) \end{aligned}$$

The above steps are justified as follows: (11) follows directly from (10); (12) and (13) are similar to (7)–(9). Although the outage expression in (13) cannot be solved in an exact closed form, a simple asymptotic solution can be derived at high SNR, leading to [9, Sec. IV-A]

$$P^{\text{out}}_{\text{DF},\mathcal{S}} \approx \frac{A_{|\mathcal{S}|+1}}{\bar{\Gamma}_{SD} \bar{\Gamma}_{F_{s_1}D} \ldots \bar{\Gamma}_{F_{s_{|\mathcal{S}|}}D}}, \quad (14)$$

where $A_{|\mathcal{S}|+1} =$

$$(-1)^{|\mathcal{S}|+1} + 2^{R_\mathrm{c}} \textstyle\sum_{n=0}^{|\mathcal{S}|} (-1)^{|\mathcal{S}|+n} \cdot {1}/{n!} \cdot R_\mathrm{c}^n \left(\ln(2)\right)^n. \quad (15)$$

Stage (2): From the assumption that the link channels are mutually independent, the probability that a given subset of the relays $F_i, \forall i \in \mathcal{S} \subseteq \mathcal{L}$, have perfectly reconstructed the source sequence—whereas the remaining relays $F_i, \forall i \in \mathcal{T} = \mathcal{L} \backslash \mathcal{S}$ have erroneously reconstructed it—can be formulated as

$$\begin{aligned} P^{\text{out}}_{F,\mathcal{S}} &= \int_{\gamma_{SF_{s_1}}=A_1}^{\infty} \cdots \int_{\gamma_{SF_{s_{|\mathcal{S}|}}}=A_1}^{\infty} \int_{\gamma_{SF_{t_1}}=0}^{A_1} \cdots \int_{\gamma_{SF_{t_{|\mathcal{T}|}}}=0}^{A_1} \\ &\qquad f_{\Gamma_{SF_2}}(\gamma_{SF_2}) \ldots f_{\Gamma_{SF_N}}(\gamma_{SF_N}) \mathrm{d}\gamma_{SF_N} \ldots \mathrm{d}\gamma_{SF_2} \quad (16) \\ &= \prod_{i \in \mathcal{S}} \exp\left(-{A_1}/{\bar{\Gamma}_{SF_i}}\right) \prod_{i \in \mathcal{T}} \left(1 - \exp\left(-{A_1}/{\bar{\Gamma}_{SF_i}}\right)\right) \quad (17) \\ &\approx \frac{(A_1)^{|\mathcal{T}|}}{\bar{\Gamma}_{SF_{t_1}} \ldots \bar{\Gamma}_{SF_{t_\mathcal{T}}}}. \quad (18) \end{aligned}$$

The above steps are justified as follows: in (16), the probability that the ith relay can perfectly reconstruct the source sequence is $\int_{A_1}^{\infty} f_{\Gamma_{SF_i}}(\gamma_{SF_i})\mathrm{d}\gamma_{SF_i}$ and the probability that the ith relay cannot perfectly reconstruct the source sequence is $\int_0^{A_1} f_{\Gamma_{SF_i}}(\gamma_{SF_i})\mathrm{d}\gamma_{SF_i}$, with A_1 being given as in (5); in (17), we use (1) to solve the integrations; in (18), a high-SNR approximation is obtained, based on the MacLaurin series of the exponential function, which leads to $\exp(-x_i) \approx 1 - x_i$ and $\prod_i (1 - x_i) \approx 1$ for $x_i \to 0$.

Stage (3): Finally, the OP of a conventional DF multirelay scheme is assessed $\forall \mathcal{S} \subseteq \mathcal{L}$ as

$$P_{\mathrm{DF},N}^{\mathrm{out}} = \textstyle\sum_{\forall \mathcal{S} \subseteq \mathcal{L}} P_{\mathrm{DF},\mathcal{S}}^{\mathrm{out}} \cdot P_{F,\mathcal{S}}^{\mathrm{out}} \tag{19}$$

$$\approx \sum_{\forall \mathcal{S} \subseteq \mathcal{L}} \frac{A_{|\mathcal{S}|+1}}{\bar{\Gamma}_{SD}\bar{\Gamma}_{F_{s_1}D} \ldots \bar{\Gamma}_{F_{s_{|\mathcal{S}|}}D}} \cdot \frac{(A_1)^{|\mathcal{T}|}}{\bar{\Gamma}_{SF_{t_1}} \ldots \bar{\Gamma}_{SF_{t_{\mathcal{T}}}}}, \tag{20}$$

where $P_{\mathrm{DF},\mathcal{S}}^{\mathrm{out}}$ and $P_{F,\mathcal{S}}^{\mathrm{out}}$ are given in (13) and (17), and their approximations, in (14) and (18), respectively.

VI. Channel Usage

In DF-IE the relays may forward erroneous sequences to the destination, whereas in conventional DF the relays discard such sequences. Thus, the DF-IE scheme achieves a lower OP at the expense of a higher channel usage, in comparison to the conventional DF scheme. In this section we quantify this extra channel usage. To this end, we define a channel use ratio, $R_{\mathrm{cu},N}$, as the average channel use of conventional DF over the average channel use of DF-IE, obtained as

$$R_{\mathrm{cu},N} = \underbrace{\frac{1}{N}}_{\text{DF-IE}} \cdot \underbrace{\Bigg(\underbrace{1}_{\Pr[S \text{ transmits}]} + \sum_{\forall \mathcal{S} \subseteq \mathcal{L}} |\mathcal{S}| \cdot \underbrace{P_{F,\mathcal{S}}^{\mathrm{out}}}_{\Pr[F_i \text{ transmits}, \forall i \in \mathcal{S}]} \Bigg)}_{\text{DF}}. \tag{21}$$

The above expression is justified as follows. For DF-IE, the average channel use is N, since in this scheme all relays constantly forward the received source sequence to the destination. In contrast, for conventional DF, any group of $|\mathcal{S}|$ relays forwards messages to the destination with a certain probability $P_{F,\mathcal{S}}^{\mathrm{out}}$ given as in (17), $\forall \mathcal{S} \subseteq \mathcal{L}$. In particular, at high SNR, $R_{\mathrm{cu},N}$ approaches one, because so does the probability that all relays perfectly recover the source sequence.

VII. Numerical Results and Discussion

In this section, we illustrate the derived outage probabilities and channel use ratios. The OP of DF-IE is assessed via Monte-Carlo simulation—or, equivalently, via numerical integration, from (9); the OP of DF is assessed in an asymptotic fashion, from (20), as well as via Monte-Carlo simulation—or, equivalently, via numerical integration, from (13) and (17) into (19). The channel use ratio is assessed analytically, from (21). We assume a binary phase-shift keying modulation and a channel-code rate of $1/2$, so that $R_{\mathrm{c}} = 0.5$. Moreover, we assume the average channel power gain of a given link equals $d^{-\eta}$, with d denoting the link distance and η being the path-loss exponent. For illustration purposes, we consider $\eta = 3.5$ and that all relays are located halfway between source and destination. We define an average system transmit SNR as ${}^{P_T}/_{N_0}$, where P_T is a total amount of power equally allocated among the source and all relays.

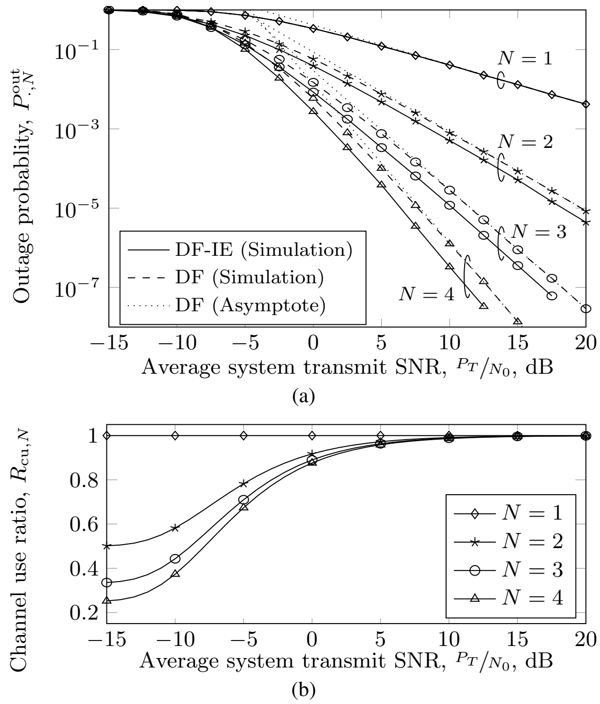

Fig. 2: DF-IE vs. conventional DF: (a) outage probability; (b) channel usage.

Fig. 2a depicts the OP of the DF-IE (exact) and conventional DF (exact and asymptotic) schemes versus the average system transmit SNR. Fig. 2b depicts the corresponding channel use ratio. In the examples, we consider the use of no, one, two, and three relays, i.e., $N \in \{1, 2, 3, 4\}$. The figures attest that DF-IE outperforms conventional DF in terms of OP, becoming more advantageous as the number of relays increases. On the other hand, such an improvement requires an extra cost in terms of channel usage, mainly at low SNR.

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